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International Journal of Heat and Mass Transfer 49 (2006) 4978–4986

International Journal of HEAT and MASS TRANSFER

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A comprehensive analysis of conduction-controlled rewetting by the Heat Balance Integral Method

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Received 17 November 2005; received in revised form 21 April 2006
Available online 21 August 2006

Abstract

A two region conduction-controlled rewetting model of hot vertical surfaces with a constant wet side heat transfer coefficient and negligible heat transfer from dry side is solved by the Heat Balance Integral Method (HBIM). The HBIM yields a simple closed form solution for rewetting velocity and temperature distribution in both dry and wet regions for given Biot numbers. Using this method it has been possible to derive a unified relationship for one-dimensional object and two-dimensional slab and rod. The effect of convection is expressed by an effective Biot number whose exact value depends on the geometry and process parameters. The solutions are found to be exactly the same as reported by Duffey and Porthouse [R.B. Duffey, D.T.C Porthouse, The physics of rewetting in water reactor emergency core cooling, Nucl. Eng. Des. 25 (1973) 379–394], Thompson [T.S. Thompson, An analysis of the wet-side heat transfer coefficient during rewetting of a hot dry patch, Nucl. Eng. Des. 22 (1972) 212–224] and Sun et al. [K.H. Sun, G.E. Dix, C.L. Tien, Cooling of a very hot vertical surface by falling liquid film, ASME J. Heat Transfer 96 (1974) 126–131; K.H. Sun, G.E. Dix, C.L. Tien, Effect of precursory cooling on falling-film rewetting, ASME J. Heat Transfer 97 (1974) 360–365]. Good agreement with experimental results is also observed.

Keywords: Rewetting; Quenching; Integral method; Conduction controlled; Effective Biot number

1. Introduction

Rewetting of hot surface is a process in which a liquid wets a hot surface by displacing its own vapour that otherwise prevents the contact between the solid and liquid phases. When a liquid comes in contact with a sufficiently hot surface a vapour blanket is formed that separates the liquid from the surface. As the surface cools, the vapour film collapses and the surface–liquid contact is re-established. The phenomenon of non-wetting of a hot surface by liquid and its subsequent wetting due to the collapse of the intermediate vapour film has been long observed under different contexts. This phenomenon is of practical importance in the controlled rewetting in nuclear reactors during emergency loss of coolant, cryogenic systems, metallurgical processes and space station thermal control. During a postulated "loss of coolant" accident, the hot core has to pass essentially through a rewetting phase. A delay in effective cooling may result in oxidation, ballooning and melting of the fuel element and other catastrophes. This has generated immense interest in studying rewetting through both theoretical simulation [1–6] and experimental observation [1,7–9].

Falling film rewetting for several vertical geometries such as plates [5,10], rods [11–13] and tubes [14] has been modeled by a number of researchers. In general, in all models, a moving rewetting front that divides the solid into two distinct region is considered. Most of the models also consider a constant rewetting velocity that reduces the problem into a quasi-static one.

Initial efforts were made to formulate one-dimensional conduction models [1,2] that are reasonably successful in correlating rewetting phenomena at low Peclet number. Tien and Yao [10] presented the asymptotic solutions of

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doi:10.1016/j.ijheatmasstransfer.2006.05.038

Nomenclature

A, B, D	constants defined in text
a	radius of cylinder, m
Bi	Biot number $\frac{h\delta}{K}$ and $\frac{ha}{K}$ in one-dimensional and two-dimensional case, respectively
C	specific heat, J/kg °C
HBIM	Heat Balance Integral Method
h	heat transfer coefficient, W/m^2 °C
K	thermal conductivity, W/m °C
M	effective Biot number
N	magnitude of precursory cooling
Pe	dimensionless wet front velocity $\frac{u\delta\rho C}{K}$ and $\frac{\rho Cua}{K}$ for one-dimensional and two-dimensional case, respectively
T	temperature, °C
T_O	wet front temperature that corresponds to the temperature at the minimum film boiling heat flux, °C
T_S	saturation temperature, °C
T_W	initial temperature of the dry surface, °C
t	time, s
u	wet front velocity, m/s
$\bar{x},\bar{y},\bar{r}$	length coordinates, m
x, y, r	dimensionless length coordinates

Greek symbols

α, β, γ	constants defined in text
δ	wall thickness, m
θ	non-dimensional temperature defined in Eq. (3)
θ_1	non-dimensional temperature parameter defined in Eq. (3)
$\bar{\theta}$	non-dimensional temperature integral defined in Eq. (7)
θ_i	non-dimensional surface temperature
ρ	density, kg/m^3

Subscripts

0	quench front
l	liquid side
v	dry side
+	evaluated at an infinitesimal increment of distance
−	evaluated at an infinitesimal increment of distance
1, 2, 3, 4	separation constants

a two-dimensional conduction model which clearly demonstrates the different physical pictures for the cases of high and low coolant flow rates and also establishes the limitation of one-dimensional model for high values of Peclet number and Biot number. A variety of techniques have been used for solving two-dimensional conduction models for falling film rewetting. Some of the important studies are elaborated below.

Because of mathematical difficulty, most two-dimensional analyses are either approximate or numerical ones. The solution by separation of variables to rewetting problem was first considered by Duffey and Porthouse [4]. They retained only the first term in the series solution. However, Coney [5] reported that using a small number of terms in a series yields inaccurate results due to slow convergence of the series. An approximate solution to the same model for a cylindrical rod was presented by Blair [11]. Tien and Yao [10] first applied the Wiener–Hopf technique to a two-dimensional rewetting problem of a rectangular slab, while an exact solution to the same problem was presented by Castiglia et al. [15], employing the method of separation of variables. Numerical solutions of conduction controlled rewetting were provided by Satapathy et al. [14,16], Thompson [17], and Raj and Date [18] by using the finite difference technique.

A solution to the rewetting problem was obtained by Olek [19] by considering rewetting as a conjugate heat transfer problem. In this method the heat transfer coefficient need not be specified unlike other rewetting models but it may be obtained as a part of the solution. Recently, a solution to the rewetting problem was obtained by Dorfman [20] by considering a transient rewetting process. It was observed that the transient cooling process is governed by a dimensionless parameter called the Leidenfrost number, expressed as the ratio of Biot number to square of Peclet number.

HBIM is one of many semi-analytical methods used to solve conduction problems [21–23]. This is analogous to classical integral technique used for fluid flow and convective heat transfer analysis [24]. This technique is simple yet it gives reasonable accuracy. HBIM has mostly been employed for a variety of Stefan problems involving one-dimensional conduction problems. However, efforts have also been made to employ HBIM for two-dimensional problems [23]. Sfeir [25] and Burmeister [26] has successfully employed this technique for analysis of two-dimensional fins. Recently some modifications [27,28] have been suggested to the basic HBIM. Rewetting of hot solid exhibits some similarity with the classical Stefan problem. Both are moving boundary problems and in both the cases the solution space can be divided into two domains with a strong temperature gradient at the interface. However only a single investigation [29] has yet been reported on the application of HBIM for rewetting analysis. This has motivated application of HBIM for a comprehensive study of conduction controlled rewetting. In the present investigation three different cases of rewetting have been considered. A generalization of all three cases is possible by application of HBIM. The results obtained from HBIM matches with the earlier analytical results and depicts good agreement with experimental results considering a wide range of parametric variation.

2. Theoretical analysis

2.1. Assumptions

Fig. 1 schematically depicts the falling film rewetting of a two-dimensional slab and a rod of infinite length. The following assumptions are made for the analysis.

(a) The wet front travels with constant velocity, and the end effects are neglected. This reduces the problem to a quasi-steady one [2–4].
(b) Based on the above assumption, one can conceive of a two region (wet and dry) conduction problem with a sharp temperature gradient at the interface.
(c) For the wet region constant heat transfer coefficient (h) is assumed and for dry region, adiabatic condition with ($h = 0$) is assumed (i.e. precursory cooling has been neglected). It is justified for low flow rates but for high flow rates, neglecting the precursory cooling may result in an under prediction of the rewetting velocity.
(d) Behind the wet front, the surface temperature drops sharply and approaches liquid saturation temperature T_S.

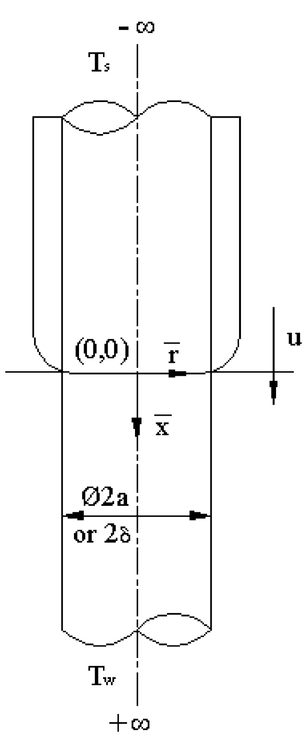

Fig. 1. Schematic diagram of top flooding of hot surfaces of a two-dimensional object.

(e) The effect of secondary factors such as system pressure, surface finish, etc. are either negligible or they do not affect the rewetting temperature.
(f) As is common in most of the rewetting models, suitable values of rewetting temperature and heat transfer coefficient are taken as input parameters.

2.2. Two-dimensional formulation

The conduction equation valid both for rectangular Cartesian and cylindrical polar coordinate systems (Fig. 1) can be written in a generalized form:

$$\frac{1}{\bar{r}^n}\frac{\partial}{\partial \bar{r}}\left(\bar{r}^n\frac{\partial T}{\partial \bar{r}}\right)+\frac{\partial^2 T}{\partial^2 \bar{x}^2}=\frac{\rho C}{K}\frac{\partial T}{\partial t},$$
$$0<\bar{r}<a,\ -\infty<\bar{x}<\infty,\ t>0$$
$$n=\begin{cases}0 & \text{for a Cartesian geometry, with } \bar{r}\equiv\bar{y}, a\equiv\delta\\ 1 & \text{for a cylindrical geometry.}\end{cases} \tag{1}$$

The quasi-steady state assumption ($\partial T/\partial t = -u\partial T/\partial \bar{x}$) yields:

$$\frac{1}{\bar{r}^n}\frac{\partial}{\partial \bar{r}}\left(\bar{r}^n\frac{\partial T}{\partial \bar{r}}\right)+\frac{\partial^2 T}{\partial^2 \bar{x}^2}+\frac{\rho C u}{K}\frac{\partial T}{\partial \bar{x}}=0,$$
$$0<\bar{r}<a,\ -\infty<\bar{x}<\infty \tag{2}$$

The following normalized variables are defined:

$$r=\frac{\bar{r}}{a},\quad x=\frac{\bar{x}}{a},\quad Pe=\frac{\rho C u a}{K},\quad Bi=\frac{ha}{K},$$
$$\theta=\frac{T-T_S}{T_O-T_S},\quad \theta_1=\frac{T_W-T_O}{T_O-T_S} \tag{3}$$

Utilizing Eq. (3), the energy equation (2) is transformed into the following form:

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial \theta}{\partial r}\right)+\frac{\partial^2\theta}{\partial x^2}+Pe\frac{\partial\theta}{\partial x}=0 \tag{4}$$

Subject to boundary conditions:

$$r=0,\quad -\infty<x\leqslant 0 \qquad \frac{\partial\theta_l}{\partial r}=0 \tag{5a}$$
$$r=1,\quad -\infty<x\leqslant 0 \qquad \frac{\partial\theta_l}{\partial r}=-Bi\theta_i \tag{5b}$$
$$x\to-\infty \qquad \theta=0 \tag{5c}$$
$$r=0,\quad 0\leqslant x<\infty \qquad \frac{\partial\theta_v}{\partial r}=0 \tag{5d}$$
$$r=1,\quad 0\leqslant x<\infty \qquad \frac{\partial\theta_v}{\partial r}=0 \tag{5e}$$
$$x\to+\infty \qquad \theta=1+\theta_1 \tag{5f}$$
$$r=1\quad x=0 \qquad \theta=1 \tag{5g}$$
$$r=1\quad (-\infty<x<+\infty) \qquad \theta=\theta_i \tag{5h}$$

Subscript l denotes the region above the quench front, and v denotes the dry region.

2.2.1. Solution procedure

In the present two-dimensional conduction problem, the axial conduction is stronger (particularly near the wet front) compared to conduction along the transverse direction. Therefore, it is decided to assume a temperature profile in the transverse direction. In the generalized form, the energy equation can be integrated as follows:

$$\int_0^1 \left[\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial\theta}{\partial r}\right)+\frac{\partial^2\theta}{\partial x^2}+Pe\frac{\partial\theta}{\partial x}\right]r^n\,\mathrm{d}r=0 \qquad (6)$$

and $\bar{\theta}$ can be defined as $\bar{\theta}=\int_0^1 \theta r^n\,\mathrm{d}r$ (7)

This gives a single ordinary differential equation valid for both wet and dry region:

$$\frac{\mathrm{d}^2\bar{\theta}}{\mathrm{d}x^2}+Pe\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}x}+\left(r^n\frac{\partial\theta}{\partial r}\right)_{r=1}-\left(r^n\frac{\partial\theta}{\partial r}\right)_{r=0}=0$$
$$(-\infty<x<+\infty) \qquad (8)$$

At this juncture it is necessary to assume a temperature profile as a function of the transverse coordinate. The success of HBIM scheme depends on the assumed profile. However, there is hardly any guideline to select the best profile. Various guess temperature profiles have been attempted in Eq. (8) and a comprehensive comparison is given in Table 1. Again for the sake of simplicity, the temperature profiles have been selected containing three unknown parameters α, β and γ. Solution of the problem assuming a generalized temperature profile for both cartesian and cylindrical geometry is elaborated below.

$$\theta=\begin{cases}\alpha_1+\beta_1 f_1(y)+\gamma_1 f_2(y) & \text{for a plane geometry}\\ \frac{\alpha_2}{G_1(r)}+\beta_2 G_2(r)+\gamma_2 & \text{for a cylindrical geometry}\end{cases} \qquad (9)$$

where $f(y)$ and $G(r)$ are functions of y and r, respectively. By using Eqs. (8) and (9) and boundary conditions (5a)–(5h) one may get,

$$\theta(x)=\begin{cases}\mathrm{e}^{\frac{Pe}{2}\left[\left(1+\frac{4M}{Pe^2}\right)^{\frac{1}{2}}-1\right]x} & (-\infty<x\leqslant 0)\\ \left[1+\theta_1(1-\mathrm{e}^{-Pex})\right] & (0\leqslant x<\infty)\end{cases} \qquad (10)$$

It is noted that the dimensionless wet front velocity, Pe, is still unknown, and is determined by using the heat flux continuity at origin.

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)_{x=0^+}=\left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)_{x=0^-} \qquad (11)$$

Combining Eqs. (10) and (11) yields:

$$\frac{\sqrt{M}}{Pe}=[\theta_1(1+\theta_1)]^{\frac{1}{2}} \qquad (12)$$

where θ_1 is a system parameter, which characterizes the dry space wall temperature far downstream with respect to temperature at the wet front.

2.3. One-dimensional formulation

The earlier solutions of conduction-controlled rewetting problems are based on one-dimensional approximation [1–4]. It is suggested [2–4] that a one-dimensional conduction model is quite successful in correlating rewetting phenomena at low coolant flow rates. We are interested in examining whether HBIM can be applicable even for this simple case. For one-dimensional conduction of heat, the governing equation and boundary conditions are as follows:

Table 1
Comparison of present analysis with the available analytical results

Source	Results	Remarks	
		Guess temperature profile	Effective Biot number M
Panel A: 2-D rectangular slab			
Duffey and Porthouse	$\frac{Bi}{Pe}=\frac{\pi}{2}(\theta_1+1)$ with $Bi<\frac{\pi^2}{4}(\theta_1+1)$ for large Peclet no.	$\alpha+\beta y+\gamma y^2$,	$Bi/(1+1/3Bi)$
Tien and Yao	$\frac{\sqrt{A}}{Pe}=\sqrt{\theta_1(\theta_1+1)}$ for small Peclet no.	$\alpha+\beta(1-y)+\gamma(y^2-y^3)$,	$Bi/(1+1/12Bi)$
Tien and Yao	$\frac{Bi}{Pe}=1.707\theta_1+\frac{1.707}{2}\theta_1^2$ for large Peclet no.	$\alpha+\beta(1-y)+\gamma(y^2+y^3)$,	$Bi/(1+17/60Bi)$
Coney	$\frac{Bi}{Pe}=1.6\theta_1(1+\theta_1)$ with $\frac{Bi}{Pe}<1$	$\alpha+\beta(y-y^5)+\gamma(y^2-y^4)$,	$Bi/(1+1/15Bi)$
Present result $\sqrt{M}/Pe=[\theta_1(1+\theta_1)]^{\frac{1}{2}}$		$\alpha+\beta(y-y^4)+\gamma(y^3-y^4)$,	$Bi/(1+1/20Bi)$
Panel B: 2-D cylindrical rod			
Blair	$\frac{Bi}{Pe}=\frac{\pi}{2}\theta_1$	$\alpha/r+\beta r+\gamma$	$2Bi/(1+1/3Bi)$
Present result $\sqrt{M}/Pe=[\theta_1(1+\theta_1)]^{\frac{1}{2}}$		$\alpha/(r+r^2)+\beta(r+r^3)+\gamma$	$2Bi/(1+7/30Bi)$
		$\alpha/(r+r^2)+\beta(r-r^4)+\gamma$	$2Bi/(1+1/9Bi)$
		$\alpha/(r+r^2)+\beta(r-r^2)+\gamma$	$2Bi/(1+1/6Bi)$
		$\alpha/(r+r^2)+\beta(r^2-r^4)+\gamma$	$2Bi/(1+1/12Bi)$
Panel C: 1-D solid			
Duffey and Porthouse	$\frac{\sqrt{Bi}}{Pe}=\sqrt{\theta_1(\theta_1+1)}$		
Yamounchi	$\frac{\sqrt{Bi}}{Pe}=\sqrt{\theta_1(\theta_1+1)}$	$D_1\mathrm{e}^{-A_1x}+D_2\mathrm{e}^{+B_1x}$ (wet region)	Bi
Semeria and Martinet	$\frac{\sqrt{Bi}}{Pe}=\theta_1$	$D_3+D_4\mathrm{e}^{-A_2x}$ (dry region)	
Present result $\sqrt{M}/Pe=[\theta_1(1+\theta_1)]^{\frac{1}{2}}$			

$$K\frac{\partial^2 T}{\partial \bar{x}^2} - \frac{h}{\delta}(T - T_S) = \rho C\frac{\partial T}{\partial t}, \quad -\infty < \bar{x} < \infty \tag{13}$$

where δ is the thickness of the slab and density, specific heat and thermal conductivity of the slab material are ρ, C and K, respectively. Employing a quasi-steady state assumption ($\partial T/\partial t = -u\partial T/\partial \bar{x}$) yields:

$$K\frac{\partial^2 T}{\partial \bar{x}^2} - \frac{h}{\delta}(T - T_S) + \rho C u\frac{\partial T}{\partial \bar{x}} = 0, \quad -\infty < \bar{x} < \infty \tag{14}$$

The following normalized variables are defined:

$$x = \frac{\bar{x}}{\delta}, \quad Pe = \frac{u\delta\rho C}{K}, \quad Bi = \frac{h\delta}{K} \tag{15}$$

Utilizing Eq. (15), the energy equation (14) is transformed into the following form:

$$\frac{d^2\theta}{dx^2} + Pe\frac{d\theta}{dx} - Bi\theta = 0 \tag{16}$$

subject to boundary conditions:

$$\theta(-\infty) = 0 \quad \theta(0) = 1, \qquad -\infty < x \leqslant 0 \tag{17a}$$

$$\theta(0) = 1 \quad \theta(+\infty) = 1 + \theta_1, \qquad \infty \leqslant x < 0 \tag{17b}$$

2.3.1. Solution procedure

In the generalized form, the energy equation (16) can be integrated as follows:

$$\int_{-\infty}^{+\infty}\left(\frac{d^2\theta}{dx^2} + Pe\frac{d\theta}{dx} - Bi\theta\right)dx = 0 \tag{18}$$

In most of the integral techniques, assumption of polynomial guess function is customary [23,24]. However, it may be noted that in the configuration being analyzed, the temperature decays (or increases slowly) away from the origin. Recently Mosally et al. [28] suggested an exponential trial function for a decay like spatial variation in temperature for a Stefan problem. They also demonstrated that if selected properly, exponential profile gives better result compared to polynomial approximation.

For the present case the following trial guess profile has been selected.

$$\theta(x) = \begin{cases} D_1 e^{-A_1 x} + D_2 e^{+B_1 x} & (-\infty < x \leqslant 0) \\ D_3 + D_4 e^{-A_2 x} & (0 \leqslant x < \infty) \end{cases} \tag{19}$$

where, A, B and D are positive constants. Using boundary conditions given in Eqs. (17)–(19) yields:

$$\theta(x) = \begin{cases} e^{\frac{Pe}{2}\left[\left(1+\frac{4Bi}{Pe^2}\right)^{\frac{1}{2}}-1\right]x} & (-\infty < x \leqslant 0) \\ 1 + \theta_1(1 - e^{-Pex}) & (0 \leqslant x < \infty) \end{cases} \tag{20}$$

It is noted that the dimensionless wet front velocity Pe, is unknown and can be determined as has been done in the two-dimensional case. Changing $Bi = M$ one gets:

$$\frac{\sqrt{M}}{Pe} = [\theta_1(1 + \theta_1)]^{\frac{1}{2}} \tag{21}$$

This is a profound result as a single expression represents the result of all the different cases analyzed with variable parameter 'M'.

3. Results and discussion

We have tried different temperature profiles for the two-dimensional slab and rod. A comprehensive summary of this exercise is presented in Table 1. Available closed form expressions from the published literature have also been incorporated in Table 1. For one-dimensional modelling, the freedom of selecting a guess profile is restricted by the asymptotic boundary condition. Though several forms of temperature profile have been tried for this case, no profile other than the reported one gives a meaningful solution. The parameter M introduced in the present work may be conceived as the sole effect of wet side convective heat transfer coefficient or the effective Biot number. In the rest of the work we have tried to depict the results for our analysis in terms of M.

Dua and Tien [30] defined an effective Biot number $\overline{Bi}$ as a function of Biot number and non-dimensional rewetting temperature and finally obtained separate relationships between Peclet number and effective Biot number for small and large Biot numbers. Most other researchers obtained analytical or semi-empirical relationships among three parameters, namely, Peclet number, Biot number and θ_1. However different functional forms were obtained. It may be noted that some of the analytical relationships given in Table 1 match exactly with our HBIM results. From a relationship as depicted in Eq. (21), some important physical aspects of rewetting may be described.

3.1. Temperature distribution in solid near quench front

The effect of effective Biot number M and non-dimensional wet front temperature θ_1 on the variation of surface temperature near the wet front are depicted in Fig. 2a and b. Both figures depict a sharp gradient of temperature at the wet front. However, the gradient increases with increase of θ_1 as well as M. θ_1 directly indicates the difference between drywall temperature and the wet front temperature. As a result, an increase of θ_1 results in a steeper temperature gradient particularly in dry side. On the other hand, a large value of M indicates a higher rate of heat removal by the progressing coolant front. This leads to a sharper temperature gradient at the point of rewetting. Similar results have also been obtained from earlier theoretical as well as experimental observations. Fig. 3 depicts the variation of Peclet number as a function of M for different values of dimensionless temperature θ_1. As Peclet number signifies the propagation velocity of quench front, it increases with M. A hot surface is quenched easily when the surface temperature is not much different from the quench temperature or when the temperature of coolant is not very close to rewetting temperature. Both these situations give

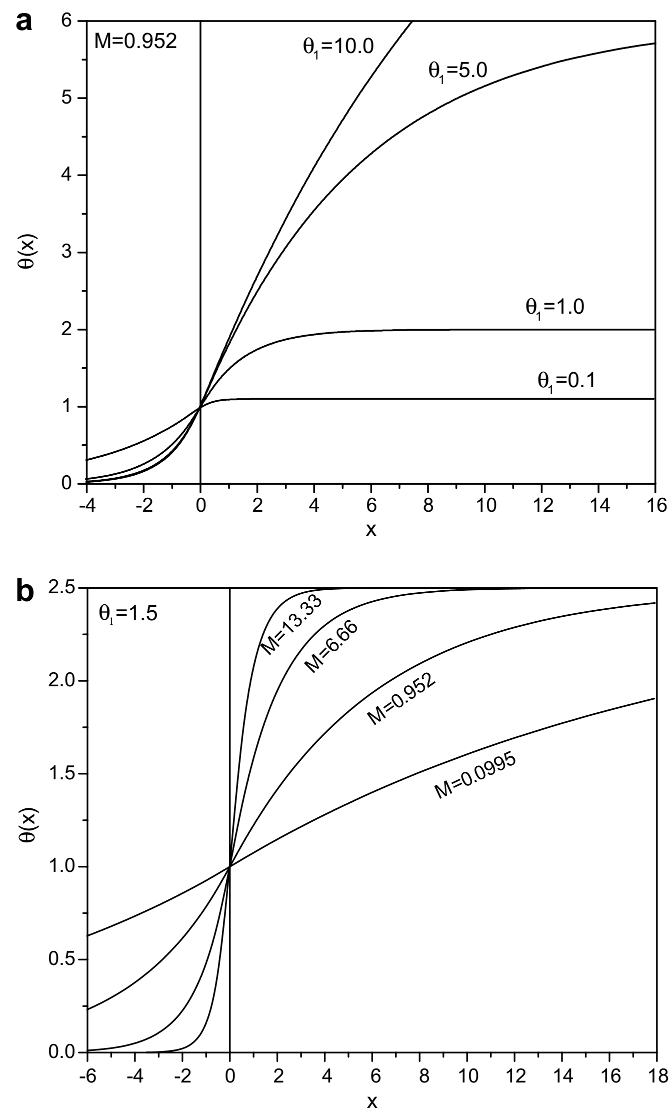

Fig. 2. Variation of surface temperature near wet front with (a) θ_1, (b) effective Biot number M.

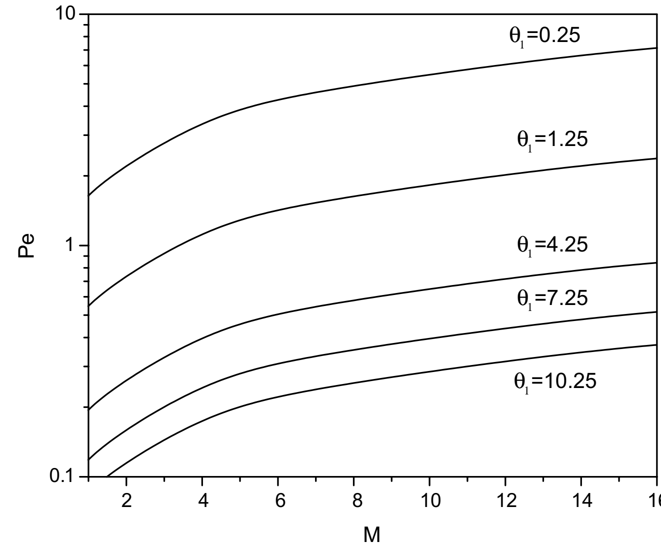

Fig. 3. Variation of Peclet number with effective Biot number M.

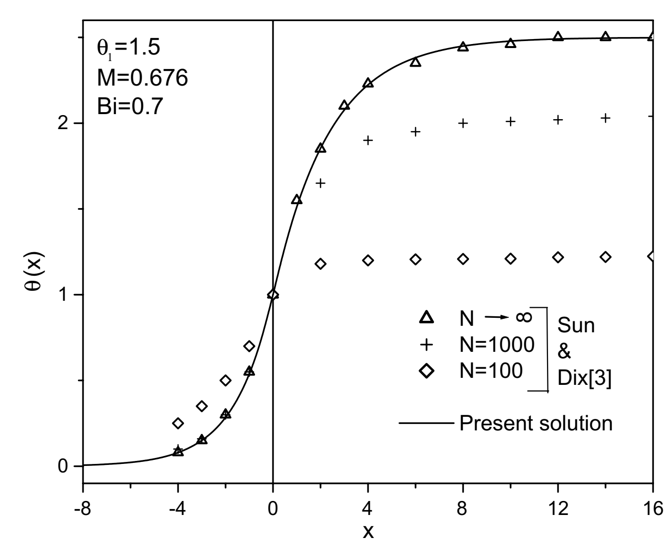

Fig. 4. Temperature profiles near the wet front for comparison with Sun et al. [3].

a small value of θ_1. Peclet number, therefore, increases with decrease with θ_1 as shown in Fig. 3.

Sun et al. [3] provided one-dimensional analytical solution for conduction controlled rewetting considering precursory cooling. To incorporate precursory cooling, they introduced a parameter N, such that $N \to \infty$ renders the heat transfer coefficient ahead of wet front to essentially zero. In Fig. 4 the present results have been compared with those obtained from the analysis of Sun et al. [3]. In the absence of any precursory cooling, the result obtained by HBIM closely matches with the analysis of Sun et al. [3].

In Table 2, theoretical prediction by other researchers have been compared with the present result for a particular value of θ_1. For the smallest Biot number ($Bi = 0.1$) all the previous analytical results matches with HBIM very closely. With the increase of Biot number the agreement weakens. But it is interesting to note that the agreement between various reported analytical results also become poor with

Table 2
Comparison among Peclet numbers using different models for $\theta_1 = 0.5$

Bi	Anderson–Hansen (2-D) and Yamanouchi (1-D)	Dua-Tien (2-D)	Tien-Yao (2-D)	Present solution $M = Bi/(1 + Bi/3)$
0.1	0.2236	0.2258	0.22	0.22
0.5	0.5	0.5244	0.4629	0.4629
1	0.7071	0.7746	0.6124	0.6124
5	1.5811	2.236	0.9686	0.9686
10	2.2361	3.873	1.0742	1.074
50	5	16.583	1.1896	1.1896
100	7.0711	32.404	1.2068	1.2068

the increase in Biot number. At the highest Biot number ($Bi = 100$) an order of magnitude difference between some of the analytical results are noted. At high Biot number, change in the local heat transfer characteristics near the

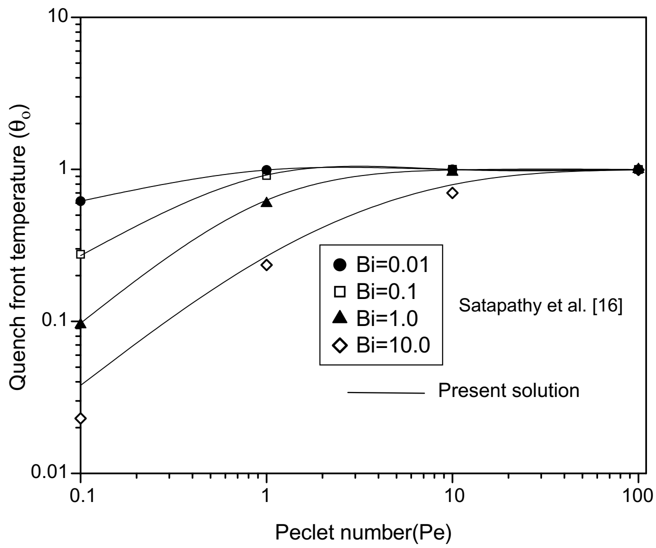

Fig. 5. Variation of quench front temperature with Peclet number for comparison with Satapathy et al. [16].

wet front and the existence of precursory cooling cannot be ruled out. Such possibilities have not been considered in the present HBIM formulation.

At this point it would be interesting to compare the result of present analysis with that obtained through numerical techniques. The quench front temperatures obtained from HBIM for a two-dimensional slab have been compared with a finite difference solution provided by Satapathy and Kar [16] for various combinations of Peclet number and Biot number as shown in Fig. 5. The present analysis not only shows an excellent matching with the numerical results in terms of trend but the quantitative agreement between the two is also remarkable.

3.2. Rewetting velocity – comparison with experimental results

As is evident from the literature survey, different theoretical models have been developed for the top flooding of slabs and rods. In all models, the conduction equation is solved treating the Biot number as a known parameter. On the other hand a host of experiments on top flooding has been conducted mostly for rods for a varied range of coolant flow rates. In a majority of the experiments water has been used as the coolant under atmospheric pressure condition. A precise estimation of the heat transfer coefficient (Biot number) has not been mentioned in these experiments. In most of the cases, Biot number values either have been guessed or approximate values have been taken for comparing theoretical prediction with experimental results [2,3,10]. Moreover, it has been argued that geometry does not have a strong effect in conduction controlled rewetting [4]. In the present analysis we are able to forward a unified three parameter relationship for conduction controlled rewetting; hence we tried to correlate the reported experimental data with the present analysis based on a parametric variation of M (effective Biot number). Results are shown in Figs. 6–9. It may be noted that the experimental data are from multiple sources and for different experimental conditions. They cover a wide variation of coolant flow rate, various coolant and drywall temperatures and different wall thicknesses. The present results are compared with the experimental results of Duffey and Porthouse [7] for a water-stainless steel pair (Figs. 6 and 7) and with Dua and Tien [8] for a nitrogen–copper pair (Fig. 9). Nevertheless, the agreement between Eq. (21) and the data set is strikingly good. This establishes the worth of the present analysis. Further, our postulation of using M as a unique parameter in the analysis of conduction controlled rewetting is strongly supported by this argument.

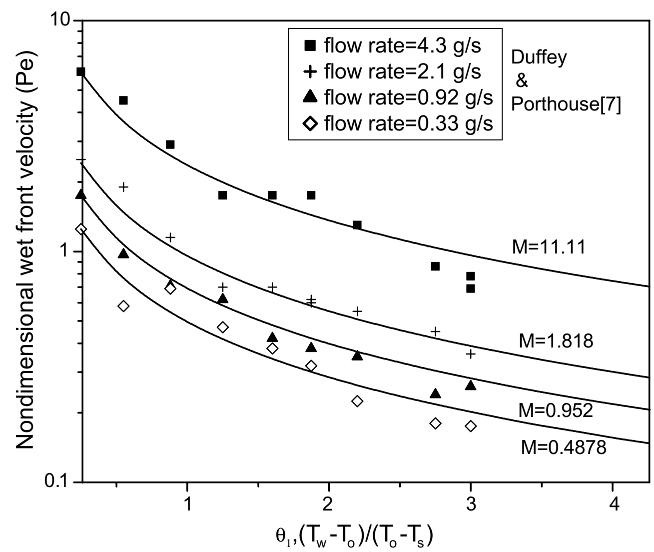

Fig. 6. Comparison of predicted wet front velocity with experimental results of Duffey and Port house [7] (wall thickness 0.05 cm).

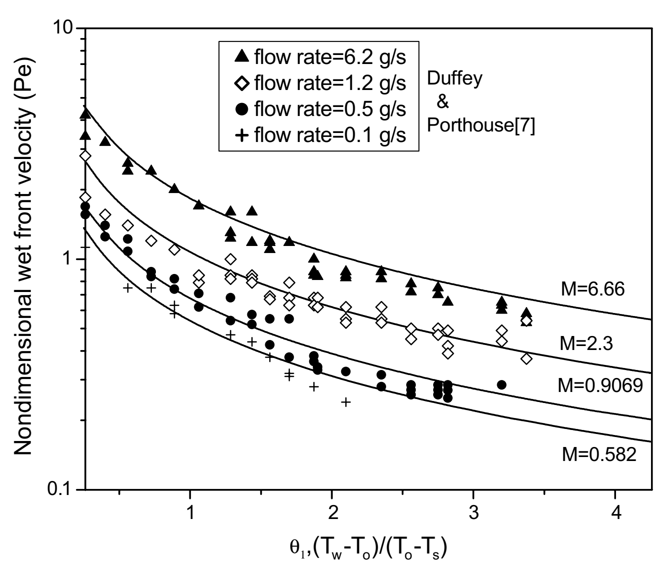

Fig. 7. Comparison of predicted wet front velocity with experimental results of Duffey and Porthouse [7] (wall thickness 0.085 cm).

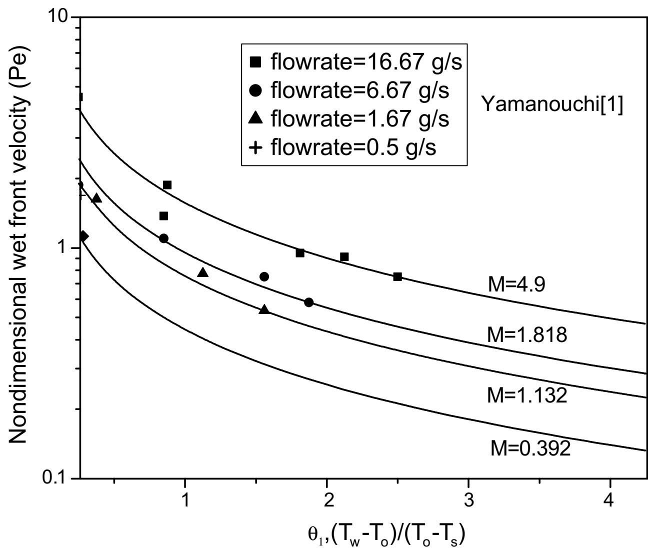

Fig. 8. Comparison of predicted wet front velocity with experimental results of Yamanouchi [1] (wall thickness 0.1 cm).

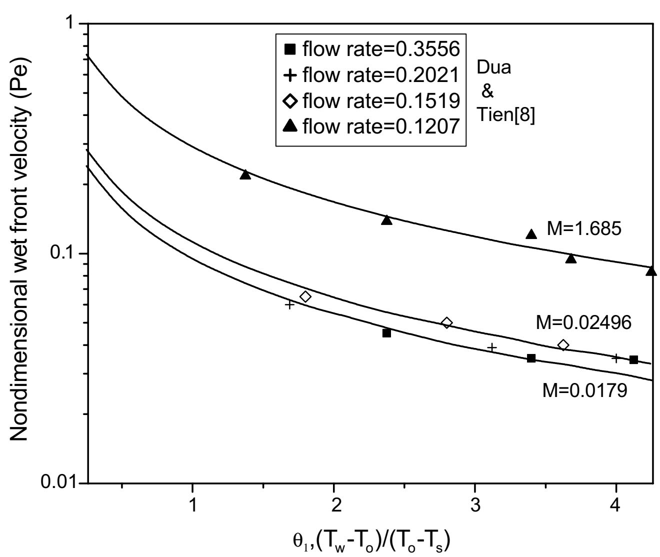

Fig. 9. Comparison of predicted wet front velocity with experimental results of Dua and Tien [8] (wall thickness 0.085 cm).

4. Conclusions

The salient conclusions of the present investigations are enumerated below:

1. HBIM has been applied for a comprehensive analysis of conduction-controlled rewetting. Three different cases comprising two-dimensional slab and rod and one-dimensional slab have been considered.
2. One of the main contributions of the present work is the introduction of an effective Biot number M. This enables to provide a generalized three-parameter relationship for conduction controlled rewetting irrespective of the geometry of the solid.
3. The published experimental results on rewetting do not report the precise values of Biot number. The present work demonstrates how different experimental results can be correlated with the analytical expression through the use of the unique parameter M.
4. Compared to the conventional numerical techniques such as finite difference and finite element methods, the present analysis is much simpler and provides analytical solution taking recourse to no special computation. Nevertheless, the quench front temperature predicted by the present method agrees very well with the finite difference results for a wide range of Biot number and Peclet number.

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